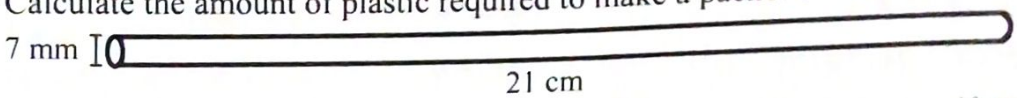
(c) A cylindrical plastic straw has a length of 21 cm and a diameter of 7 mm. Calculate the amount of plastic required to make a packet of 100 straws.

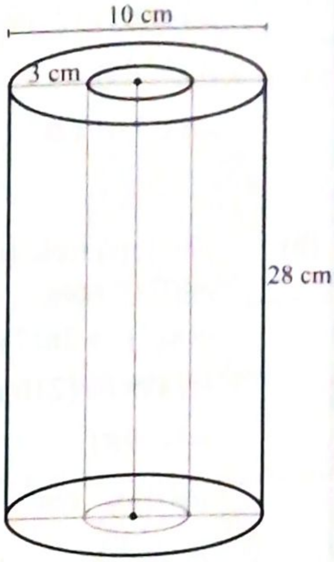


(d) 52 rectangular sheets of soft paper called roller towels are joined together and wrapped around a hollow cardboard cylinder. The dimensions of each roller towel are $275 \text{ mm} \times 220 \text{ mm}$. The roller towels and cardboard cylinder form a larger cylinder with a diameter of 10 cm and height 28 cm. The distance between the outer and inner circumferences is 3 cm.

(1) Calculate the volume of space taken up by the roller towels.

(2) Calculate the total amount of soft paper used.

(3) Calculate the amount of cardboard used to manufacture the cardboard cylinder.



The relationship between surface area and volume

Example 7

The dimensions of a cylinder A are doubled to form a larger prism B.

(a) Calculate the surface area of A.

(b) Calculate the surface area of B.

(c) Determine the ratio $\dfrac{\text{Surface area of B}}{\text{Surface area of A}}$.

(d) Compare the surface area of A and B. What do you notice?

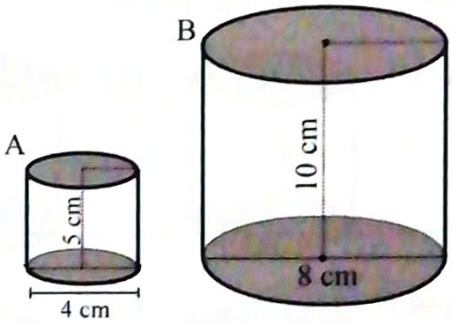
(e) Calculate the volume of A.

(f) Calculate the volume of B.

(g) Determine the ratio $\dfrac{\text{Volume of B}}{\text{Volume of A}}$.

(h) Compare the volume of A and B. What do you notice?

(i) What is the relationship between the surface area and volume if the dimensions are doubled?



Solutions

(a) Surface area of A

$= 2\pi(2)^2 + 2\pi(2)(5)$

$= (8\pi) + (20\pi)$

$= (28\pi) \text{ cm}^2$

(b) Surface area of B

$= 2\pi(4)^2 + 2\pi(4)(10)$

$= (32\pi) + (80\pi)$

$= (112\pi) \text{ cm}^2$

(c) $\dfrac{\text{Surface area of B}}{\text{Surface area of A}} = \dfrac{(112\pi) \text{ cm}^2}{(28\pi) \text{ cm}^2} = 4$ (Notice that $4 = 2^2$)

(d) Surface area of B $= (112\pi) \text{ cm}^2 = 4 \times (28\pi) \text{ cm}^2 = 4 \times \text{Surface area of A}$

(e) Volume of A $= \pi(2)^2(5) = (20\pi) \text{ cm}^3$

(f) Volume of B $= \pi(4)^2(10) = (160\pi) \text{ cm}^3$

(g) $\dfrac{\text{Volume of B}}{\text{Volume of A}} = \dfrac{(160\pi)\text{ cm}^3}{(20\pi)\text{ cm}^3} = 8 = 2^3$ (Notice that $8 = 2^3$)

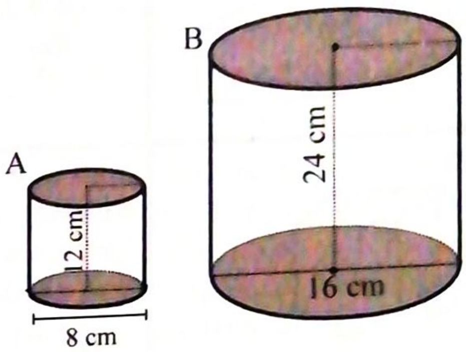
(h) Volume of $\text{B} = (160\pi)\text{cm}^3 = 8\times(20\pi)\text{cm}^3 = 8\times$ Volume of A.

(i) If the dimensions are doubled (multiplied by 2), then the surface area of B is $2^2 \times$ the surface area of A and the volume of B is $2^3 \times$ the volume of A.

EXERCISE 5

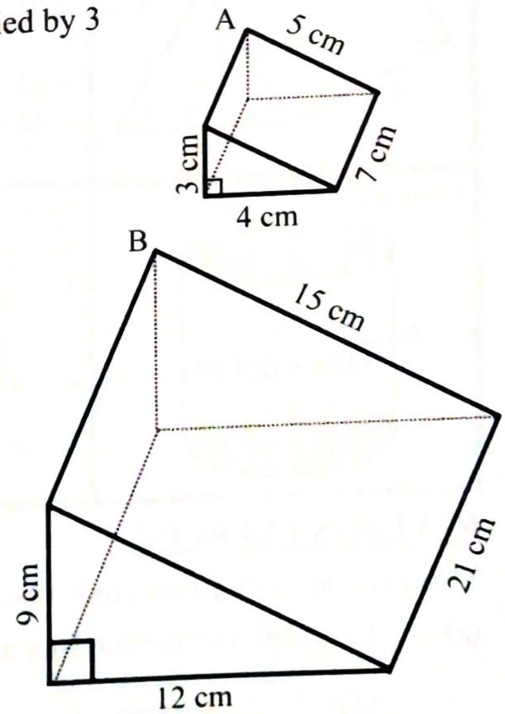
(a) The dimensions of a cylinder A are doubled to form a larger prism B.

(1) Calculate the surface area of A.
(2) Calculate the surface area of B.
(3) Determine the ratio $\dfrac{\text{Surface area of B}}{\text{Surface area of A}}$.
(4) Compare the surface area of A and B. What do you notice?
(5) Calculate the volume of A.
(6) Calculate the volume of B.
(7) Determine the ratio $\dfrac{\text{Volume of B}}{\text{Volume of A}}$.
(8) Compare the volume of A and B. What do you notice?
(9) What is the relationship between the surface area and volume if the dimensions are doubled?



(b) The dimensions of a triangular prism A are multiplied by 3 to form a larger prism B.

(1) Calculate the surface area of A.
(2) Calculate the surface area of B.
(3) Determine the ratio $\dfrac{\text{Surface area of B}}{\text{Surface area of A}}$.
(4) Compare the surface area of A and B. What do you notice?
(5) Calculate the volume of A.
(6) Calculate the volume of B.
(7) Determine the ratio $\dfrac{\text{Volume of B}}{\text{Volume of A}}$.
(8) Compare the volume of A and B. What do you notice?
(9) What is the relationship between the surface area and volume if the dimensions are multiplied by 3?



Conclusion
From the previous example and exercise, it should be clear that if the dimensions of a cube, cuboid and triangular prism are multiplied by a number k (called the scale factor), then the relationship between the surface area and volume is as follows:
Surface area of enlarged prism $= k^2 \times$ surface area of original prism
Volume of enlarged prism $= k^3 \times$ volume of original prism